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in any one department should be available in every other department.

8. Hence it should be a step toward giving geometry a wider, deeper, and more definite cultural value.

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THE SYLLABUS METHOD OF TEACHING PLANE GEOMETRY.*

BY EUGENE R. SMITH.

In estimating the values of different methods of teaching a subject, it is first necessary to consider the purposes for which the subject under discussion is taught. Geometry has a double purpose: its *facts* are of use in many vocations, and indeed, in occasional instances, in the everyday home life of man or woman; while, on the other hand, the ***methods*** and ***habits*** of correct geometrical work—the logical reasoning from known premises to their infallible conclusions—are of inestimable value to every intelligent person, even aside from their direct application to certain of the professions, such as branches of engineering. You will notice that I have made a distinction between the use of the *facts* of geometry in such a vocation as carpentry, metal working, and other trades, and the use of its methods in the scientific professions.

If there were only the facts of geometry to consider, a man might need to know that the sum of the angles of a triangle was a straight angle, but the reasons why would be of no concern to him. He might, then, carry a convenient table of geometrical facts in his pocket, and refer to it when necessary, or even memorize the comparatively short list of propositions which are of so-called "practical" nature. Why should he waste his time studying theoretical geometry? It would be nonsense.

It is only in case geometry has an important purpose, aside from the direct applicability of its facts to vocations, that the study of the proofs of geometrical propositions should have a place in our educational system. Has it such a use? I believe that it has, and that its great value is in developing the power to think along correct lines of logical thought; if properly handled, it makes the pupils think more intensely and interestedly than any other subject fitted to pupils of the same age. In saying this I do not discredit the claims of other subjects to supply mental training along logical lines. Many other subjects *contain* logic, but geometry *is logic*. It is probably the only subject suited to immature minds in which one can be expected to see the compelling reasons back of each step taken.

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* Read at a meeting of New York Section.

Granting then, that geometry has a double purpose, do we need two methods of teaching it? That is, as yet, an unsolved question; for while the second purpose of geometry applies to everyone—and it would be a pity to deprive anyone of its valuable training—yet there are many whom lack of time, on account of the requirements of earning a living, forces to forego the broader training of the subject, and to certain of these the narrower course might be of value. These cases, however, are beyond the purpose of this paper, for the pupils under discussion are, I take it, the average secondary school students.

What method will give the pupils who take the secondary school course the most efficient training with the least waste of energy? I believe most emphatically that the syllabus method, in its best form, is the answer. It seems to me that work in which the pupil follows someone's thought, without initiative of his own, should be reduced to a minimum. But, you may ask, can you expect the pupil to work out the theorems of geometry—the results of centuries of discoveries—by himself? It is this question which has done more than any other thing to retard the development of the syllabus method, for it shows an absolute ignorance of its aims and methods. I hope to answer it by the rest of this paper.

Just a word in characterization of the three methods of teaching geometry in common use:

1. The "text-book method" is the method where the author does the thinking, and the teacher and pupils follow his lead more or less blindly, according to their several abilities. The more modern texts attempt to make up for this by urging a great deal of practice in "solving originals," the originals being any geometrical truths which the author in question did not consider of sufficient importance to be placed in the text itself. What a travesty of pedagogical psychology it is! For the im-

portant truths, the ones the pupil needs to make wholly his own, he can read the author's proofs, while the unimportant ones are given him for his own training. To my mind, this solving of originals can never give to a child the sense of logic which the development of the whole subject as an investigation does; while nothing can make up to the pupil for the time spent in, as the students too often express it, "learning the proofs." It has been estimated on good authority that perhaps 75 per cent. of the pupils studying geometry from a text get the proofs largely by memory.

It is sometimes said for the text-book method that it enables the pupil to read a great number of proofs and so familiarizes him with the subject. There is no doubt that reiteration drills the memory, but I doubt very much whether reading geometrical proofs trains the logical faculties so that a pupil can apply his training to other geometrical questions.

I do not deny that a good teacher can overcome the handicap of a text-book, but I do assert that it fetters the originality of both teacher and pupil, and that only the best of teachers should dare to use one—and then with constant care lest he become a mere copyist and drillmaster.

2. The suggestive method is the method where the author leaves part of the thinking for the teacher and pupils to develop; it seems to me to be a step in advance, in spite of some objections to it which I will discuss later.

3. The syllabus method is that in which practically all the thinking is left to the teacher and pupils; the author doing little but arranging the propositions in some good order, and indicating the places which need special attention. Notice that I have included the teacher in each of the characterizations, for I consider the teacher the key to the situation.

Before going farther, I shall explain in detail just what I mean by the syllabus method. In this method, each pupil is supplied with a book, or syllabus, which contains:

1. The definitions, axioms, etc., and a fairly comprehensive discussion of them. This part is never assigned to the class for study before it has been thoroughly taken up in class work; it is used to guide the teacher as regards order and subject-matter and as a reference book for the pupils.

2. The statements of the theorems of plane geometry, arranged in some logical order.

3. A good discussion of methods; including quite frequent references to the uses of theorems and their classification, also a very few sample proofs, illustrating the different methods of attack, and developed from the standpoint of discovery.

4. A good body of exercises, preferably grouped partly under the theorems—to save the time of the teacher—and partly in general lists, to give the pupil practise in picking out the applications of the various theorems.

5. Classified summaries of the theorems.

I have already said that the teacher should develop the preliminary definition work and its most direct consequences with the class *before* assigning it to the pupils or study. Even as early as this the teacher should begin to impress upon the pupils the two great features of geometry, and therefore of a successful method of teaching it; namely, its naturalness, by which I mean the everyday common-sense way in which each step is arrived at from known data—its inevitableness, if I may use that expression; and secondly, the possibility of grouping the facts on a basis of usefulness. This should include correlations, and the study of likenesses and distinctions, and should, when possible, group theorems of the same type under one general statement.

When a pupil grasps the following statement of method, the first great step has been gained.

Any theorem must be proved by some known truth which has the same kind of a conclusion (unless it is proved by pure logic from its converse or contraposite, and this is usually very easy to determine), and so the logical way to start any proof is:

1. To list previous methods of obtaining this kind of a conclusion, that is, those of the same class;

2. To choose a method, using the "given" to eliminate those impossible of application;

3. To attempt to apply this method.

Suppose now that the preliminary definitions, axioms, and their immediate consequences have been thoroughly discussed, and that the facts so far obtained have been classified, so that the pupil knows, for instance, how to attempt to prove one line perpendicular to another, in what known cases angles are equal, etc., and the teacher starts the theorems. Does he, as so many seem to think, say to the pupil, " Now, here is a theorem, you go over in the corner, turn your face to the wall, and stay there until you have proved it?" Far from it! At the beginning most propositions, and throughout the course the most difficult ones, are fully discussed in class, and this discussion is, perhaps, the most distinctive thing in the syllabus method. The teacher is naturally the leading spirit of the discussion, but he is no longer an instructor looking down upon the pupils from the heights of superior knowledge, he is a fellow investigator, and if the teacher and the class can get into the spirit of harmonious partnership in discovery, the result is bound to be successful.

The teacher starts such a recitation with a mind absolutely open; he is not bound by the fact that a certain text gives a certain proof; he is there to find, with his pupils, the easiest and best way for them to prove that proposition. Right here I wish to interpolate the fact that I have often had two classes, doing absolutely the same work, on the same day, with no apparent influence except their individualities, find different proofs for a theorem—and each declared emphatically that the other proof was not nearly so satisfactory. The proof found by each class was the natural one for them, and no other method would have fitted the proof to the class as did the syllabus method.

But by what procedure does the teacher guide the investigation? Almost entirely by questions; and in geometry, as in law, questions which imply their own answers are barred. The teacher's first questions, if a full discussion is to be gone into, ask to what class the theorem belongs, what methods of proving this kind of theorem are known, how to draw the figure, what the hypothesis is, and which of the methods listed are manifestly impossible in the given figure. Having determined the methods which may be applicable, by this method of elimination—and there is often but one remaining—the class chooses that one which, after examination, seems to be most likely to give the desired result, and the development proceeds. After some practise in this kind of work the pupils can ask these first questions themselves. The teacher always questions concerning the "why" of the next step; he never tells the step, and says, "why" afterwards, and it is here that I believe the suggestive texts fail to accomplish the best results. They also attempt the question and answer method, but, by the very nature of such questioning, it cannot be printed and put in the hands of the pupil. Granted that the first question is correctly put, and does not imply its own answer, is not the second question almost always ***based on the answer to the first question?*** Evidently it is, and therefore the pupil could answer the first question from the second, and so on. However, most at least of the suggestive books, do not even attempt to ask questions which do not imply the answer, but get the form of the discovery method, without its intent, by practically stating the fact, and asking "why" afterwards, or else by giving the proof in a more or less disguised form. Let me illustrate:

Theorem—"The angles opposite the equal sides of a triangle are equal."

Book A, a suggestive text, gives the figure, with the auxiliary line, the hypothesis and conclusion in full, and goes on as follows:

"Suggestion 1. Let AM be drawn to represent a bisector of $\angle A$, and be extended until it meets BC, at M."

It does not even say "Why" after this. The construction is, as a matter of course, told to the pupil; of course, he could not find out what line to draw! Did you ever have a pupil say to you, after being shown some construction line in an original, "Of course I see the proof now, but how would I ever think of drawing that line?" The introduction of auxiliary lines is, as we all know, the key to a good share of the proofs, and only in the most exceptional cases, if ever, should the pupil be *told* what line to draw. The intersection of the bisector with the base is, of course, ignored. This would naturally come up for discussion in the syllabus method, as would the possibility of adding more conditions to the drawing of the line.

"Suggestion 2. Compare $\triangle ABM$ with $\triangle ACM$," and the authority is referred to. Suggestive indeed! "compare" the triangles, when the only thing done with triangles so far is to prove them congruent, and the reference even tells which theorem will do that.

"Suggestion 3. Compare $\angle B$ with $\angle C$," and the reference again!

"Therefore— "

Now I will admit at once that the pupil has thought a little for himself, but has he learned anything about how to prove a theorem? Does he know why the angle was bisected, and why the triangles were proved congruent? I believe not, except in so far as he sees it for himself, for certainly the suggestions did what the old style text has always done—taught him the proof *not how to discover the proof.*

Book B, another suggestive text, does not give the figure, but the hypothesis and conclusion are given in full, in terms of the letters of the supposed figure. Rather an evasion, though it forces the pupil to draw his own figure, and in so far, is an improvement.

"Suggestion 1. Draw a line bisecting $\angle A$, and meeting BC at M."

"Suggestion 2. Compare the $\triangle$s ABM and ACM." No authority referred to.

"Suggestion 3. Compare $\angle$s B and C, using scholiums under Th. VI."

"Conclusion."

Is it very much better? Would you not say that the function of the pupil was to look up omitted references rather than to do any of the thinking?

Now consider the syllabus method of attacking the same theorem:—the pupils would decide that angles were to be proved equal, and would list the known ways of proving angles equal. They would discover that no way could apply to the figure except showing that the angles were corresponding parts of congruent triangles. To form two triangles, some line must be drawn, and it must be drawn so that $\angle B$ and $\angle C$ are in different triangles, as are the given equal lines AB and CA. Naturally the line is drawn from A. But there are several ways of determining a line; which should be used? That one which will most readily prove the triangles congruent. As any line drawn from A will make two equal sides, the included angle is the obvious necessity, and so the angle is bisected.

This is only a rough and condensed report of the discussion, which is participated in by the whole class, but it serves to show that the reason for the step is studied before the step is taken. By this method the pupil knows why he did each thing, in other words, he is learning the method of proof rather than the proof itself; *he* is reasoning—not the teacher nor the author.

Let me ask this "How many of you who use a text, suggestive or otherwise, would consider it a fair question to ask your pupils why the auxiliary lines were used in any of the theorems? Would the pupils consider such a question as unheard of, or would they take it as a matter of course?" I know that some of you ask such questions, and in fact do all things which the syllabus method does, but if so, why the text, which can only cripple your efforts?

I could multiply examples like this theorem almost indefinitely, but it is only necessary for you to turn to any text, and read it in the light of what I have just mentioned, to see that I have not chosen an isolated example, but rather a simple and typical one.

But the syllabus method does not develop all work by class discussion, it asks the pupils to work out very many of the propositions with little or no assistance, even from questioning. This is sometimes done in class, where each pupil works along on the succession of propositions at a pace governed by his ability. The teacher is there to ask the illuminating question when the difficulty becomes too great—to often refuse to ask or answer when the pupil should conquer entirely by his own efforts, and in fact, by wise guidance and enthusiastic leadership to carry the efforts of all the members of the class to some accomplishment.

Again, certain theorems are assigned for work at home, and if the theorems are well chosen, and the pupils have been well trained in the methods of attack, the larger part of the class will be able to find the proofs.

It is of course true that some of the texts omit certain proofs altogether, and in so far, they are following the syllabus method. They have not, however, given the student the proper training in attack, and it can be no wonder if the pupils do not do any too well with such theorems, unless the theorems are quite simple ones.

But to go on with the method. After having a proof, a pupil may, or may not, be required to write it out in a predetermined logical form. If he does write it out, he hands it in to the teacher, who returns it with indicated corrections. The pupil then makes the corrections, and files the proofs in a notebook, which eventually becomes a complete geometry built up by the pupil. It is a well-known fact of psychology that a thing becomes one's own largely in the degree in which it results from one's own effort. I think this needs no comment.

The matter of requiring a pupil to keep a notebook of theorems should be determined by the conditions under which the teacher is working. A teacher who has a crowded schedule and large classes would probably sacrifice more in strength and inspiration in examining such books than would be gained by the pupils in writing the proofs. On the other hand, if the conditions are such that the teacher can do the work, the writing out of the theorems in definite concise form is a valuable training. I might say that I have always required my pupils to keep notebooks, but my classes have never exceeded twenty-eight, nor has my schedule been an overburdening one. The system is in just as successful use in some schools which require no notebooks, as in those which insist on that feature.

I wish to emphasize again that the pupils remember the theorems by classes, and by their relations to each other. To bring in psychology once more, a new concept is hard for the mind to grasp, but an addition to an established group of concepts is much more readily made part of one's knowledge.

Another feature of the syllabus method is almost daily cross-questioning, or oral review. A few typical questions follow:

What ways of proving lines parallel do you know? If you wished to prove two line sects equal, what way would you think of first? Why? Give a brief outline of a proof of the theorem for the sum of the interior angles of a polygon. A second proof. Why are triangles used? Which method of dividing the figure into triangles do you consider better and why? What is the theorem good for? (After having considered the exterior case also.) If the sum of the interior angles is five times the sum of the exterior angles, how many sides has the polygon? How would you state the converse of the interior angle theorem? the contraposite? Does either give a new theorem? etc.

You may say that such questions can be, and are, asked in any method. Yes, but such discussion is the natural outcome of the syllabus method, while it is more or less of a graft on any other. The moment the text-book is discarded, oral and individual work must take the place of the book, and so these helpful and interesting reviews and oral tests are a natural outgrowth of the method. An interesting variation is the " spelling match " with geometry questions instead of words to spell.

I have given an idea of what the syllabus method is; now I will answer a few questions and criticisms which are frequently made.

"Doesn't it take more time to cover the subject?" No. The first part takes quite a good deal longer, for the pupils must become used to the methods before very rapid progress can be made, the last part takes very much less time. I think considering amount of work covered in the whole course, more can be done by this method.

"Do you have time to do originals?" Hundreds of them—not to speak of the fact that the pupils are doing work which is largely original all the time. The originals also are done in groups when an important topic is to be studied; for example, the teacher may say, "To-day we will study the trapezoid; let us base its discussion on that of the triangle. Can you suggest any triangle theorems which could be adapted to trapezoids?" The pupils suggest certain ones, and they are investigated. In the course of the discussion, the auxiliary lines which are of most value in trapezoids are discovered, and at the end of the lesson, the pupil has a better idea of how to attack a question relating to trapezoids than he could obtain by a very large number of isolated exercises.

"Can it be used with large classes?" The questioning method of developing theorems is especially adapted to large classes, for it gives many opportunities for recitation—enough so that all can take part—and it insures that each pupil shall, to some degree at least, think for himself. As I have already said, I have never had a geometry class of over twenty-eight, but others have used the method successfully with large classes, and under none too good conditions.

"Is it not harder for the teacher?" Yes, I think it is. If you are looking for an easy time, avoid it; if for results, try it. The first thing a teacher trying the method discovers is that he doesn't know as much geometry as he supposed, for new avenues of research open all round him from day to day. That means additional work and thought for a time, but all in the line of professional advancement.

If notebooks are used, the examining of the written theorems takes time. True, that time decreases as the teacher becomes accustomed to the method, but it is never negligible.

As I have already said, there are many places where the teacher should not require this notebook work, and in such cases, a slight variation in the method would be used; for example, the pupils might be allowed to make condensed notes on the theorems as developed in class, and write out only the ones assigned for home preparation.

"Can a teacher who has had no special training use the method?" If a teacher can see geometry from the standpoint of a pupil, and can conduct the recitation as a fellow worker, he is on the highroad to success. On the other hand, one cannot have too thorough a knowledge of geometry and its allied branches, for it all helps to a more intelligent guidance of the class. Perhaps as important a requisite as any is the ability to determine the correctness of a proof.

"Do pupils, taught by this method, get into college?" With many of you this would be the supreme test. They do, and they not only get in, but they stay in—which after all is of some importance—and they do good college work because they have been taught to think.

"Do the pupils really discover the proofs for themselves?" There is hardly a theorem in plane geometry—I am not sure that there is one—which some of my pupils have not worked out for themselves. Many times a theorem which I have decided to take up in class is worked out beforehand by some ambitious students. My answer to the question of expecting the pupils to rediscover all the geometry of the ages is: 'No, I do not expect it, neither do I ask it, but nevertheless I get it in varying degrees; and, at any rate, *the pupils are thinking, they are reasoning, and their logical faculties are developing.*"

"Does it not encourage the pupils to cheat, either by using a text, or by copying notebooks?" I think not. Some pupils will be dishonest by any method, but if the method is properly worked out, the average pupil will not be dishonest. Aside from any moral standpoint, the pride in accomplishment and the pleasure that the average person takes in a puzzle, will, if properly stimulated, reduce dishonesty to a minimum. I have never found this worth considering, and I think other teachers who use the method will agree with me.

Let me give one more example of development, and with a word of conclusion, I will be through.

Take the division of a line sect internally in mean and extreme ratio. Do your pupils ever squirm when it comes along? But it is a perfectly natural proof, and if discovered in the following manner is not likely to make trouble. Let AB be the given sect, P the required point so that $AB/AP = AP/PB$. Of the four terms of this proportion, how many are known? (one, AB). If we could change the form of the proportion so that there were more known terms, would you consider it an advantage? (The pupils grasps the idea that such a change would add to his knowledge concerning the required proportion, and answer "Yes.") Now look at the figure and see if there is any relation between the terms which could be used to get a known term from unknown terms. ($AP + PB = AB$.) Is there any way to get this form in the proportion? (By composition). The proportion is then written $AB + AP/AB = AB/AP$, and it is noticed that the known term AB is used three times, while there is but one unknown term AP which is used twice. From which term must we work? (AB.) What is seen about AB? (It must be a mean.) What theorems do you know in which a sect is the mean between two others? (The students state them.) Where then ought the sects AB, and AP and $AB + AP$ to lie in the figure? (Students suggest, perhaps, the altitude to the hypotenuse of a right triangle and the sects of the hypotenuse.) A figure is drawn freehand so that the required parts are seen in the desired position. In constructing this figure, where must we start? (With the known line AB, by drawing it perpendicular to an indefinite line, say at K.) What fact about a right triangle is likely to be of assistance in constructing this right triangle? (It can be inscribed in a semi-circle.) Is anything besides the altitude known? (Part of the hypotenuse.) From what point must it be cut off? (From K.) Why? (Only known point.) The point is called L, and it is easily seen that the unknown AP must be added to each end, from K and from L, to complete the hypotenuse, and that therefore the midpoint of KL is the center of the required circle, while the radius must be the sect from this midpoint to the other end of the altitude. The circle is then constructed, and the required sect AP is found. A similar, and no harder demonstration would be found by using the tangent; I give this one because pupils so often suggest it. I leave it to you as teachers whether such an analysis does not present the subject to the pupil in a way likely to interest and benefit him.

In conclusion, I wish to say that I am an enthusiast, but not, I hope, a crank. I try to make my pupils think rather than discover, for thinking is of more importance than the mere discovery of theorems, which should be a means, not an end. I do not claim that this method is a panacea for all ills. The slow pupil is still slow, the careless one is still careless, and sometimes the uninterested are still hard to arouse, but in each of these classes I believe that a little more of the divine fire shows. If there is any way to make geometry a student's holiday, I have yet to find it, but by this method, it is at least a live, businesslike, and commonsense subject, and the pupils know that good judgment and sound logic can overcome all its difficulties.

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INTERESTING WORK OF YOUNG GEOMETERS.

By J. T. Rorer.

I.

Although that part of geometry commonly studied in our schools has been so thoroughly worked over year after year for so many centuries, nevertheless new and interesting theorems are occasionally discovered. The following theorems which are not found in the text-books commonly used and which probably have not appeared in the journals, were discovered by a second-year high school student.

1. If the segments of the altitudes of any triangle between the vertices of the angles and the orthocenter be perpendicularly bisected, the lines when produced will form a new triangle congruent to the original triangle; also,

2. The orthocenter of the first triangle is the circumcenter of the second, and conversely; also,

3. If the adjacent vertices of the two triangles are connected, an equilateral hexagon having its opposite sides parallel, will be formed.

The above theorems were suggested to the mind of the stu-